ChE 494-598: Introduction to System Identification
Course Overview

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Course Objectives

• Present fundamental background to allow students to make judicious choices of design variables in system identification.

• Provide hands-on exercises that will give students a working feel for the course topics. The System Identification toolbox in Matlab will be the program of choice; we will also examine some novel interactive software tools currently under development.

• Provide a glimpse of cutting-edge identification research at Arizona State and other academic institutions around the world.

Motivation

• In many real-life situations it is important to understand how outcomes of interest vary over time as a result of changes in independent variables (some of which can be manipulated over time). Some examples:
  - How does changing reagent flow change product composition in a chemical reactor?
  - What resonant modes are present in an aircraft wing under flutter?
  - How does changing demand affect inventory levels across a supply chain?
  - How does changing weekly exercise time affect body mass index?
  - How does changing naltrexone dosage increase adherence in an alcoholism treatment program?
System Identification

- Identification is the determination of on the basis of input and output, of a system within a specified class of systems to which the system under test is equivalent.” - Lofti Zadeh, 1962.
- System identification focuses on the modeling of dynamical systems from experimental data.

System Identification - Continued

- Broad field with applications in many areas of science and engineering; among these:
  - Providing the dynamical models that are required to build closed-loop control systems. System identification is widely recognized as the most expensive task in advanced control system implementation projects.
  - Simplifying complex simulation environments into models of manageable form. Examples of these simulation environments include discrete-event simulation and agent-based modeling.

Binary Distillation Column

- Staged separation device that separates components in a liquid mixture by taking advantage of their difference in boiling points.

Shell Cumene-Acetone Column Data

- Reflux flow is manipulated to obtain a response in overhead temperature. How can one tell that a significant disturbance is present in this data set?
The control problem is to keep the temperature constant at four points in the wafer, by adjusting the power to four sets of lamp banks.

- displays a highly nonlinear and stochastic response;
- demand models need to be identified as well.
Wing Flutter Example

- artificial mechanical vibrations (top) introduced to a wing at certain flight conditions; filtered output response (bottom).

Other Challenging Areas

- Biological Systems
  - Applications in systems biology and bioinformatics.

- Economic/financial systems
  - modeling economic indicators and financial instruments (e.g., the stock market).

- Behavioral/social systems
  - prevention and treatment of chronic, relapsing disorders (such as alcoholism, smoking, and drug abuse).

Stages of System Identification

- Experimental Design and Execution
- Data Preprocessing
- Model Structure Selection
- Parameter Estimation
- Model Validation
System identification is an inherently iterative procedure; however, some iterations are more expensive (and demanding) than others. Construct the experiment and collect data. Should data be preprocessed? Polish and present data. Choice of model structure. Fit the model to the data. Validate the model. • Statistically • Physically Can the model be accepted? Model processed data. Data not OK Model structure not OK. No Yes

Parameter estimation lies at the heart of system identification, but its effectiveness is highly dependent on the results from other steps in the process, particularly experimental design and execution.

Understanding the various system identification methods and their design variables in terms of their influence on bias and variance,

Effective use of a priori knowledge regarding the system to be identified, and the performance requirements of the intended application (e.g., simulation, prediction, closed-loop control).

"the classical statistical approach," per Lennart Ljung (Professor of Electrical Engineering at Linköping University, Sweden, and creator of the System Identification Toolbox in Matlab)

Skill-level issues: many system identification methods assume that the user has extensive background in statistics, signal processing, discrete-time systems, and optimization.

Large number of design variables.

Process operating restrictions make identification one of the most time-consuming tasks in advanced control implementation projects.

Let us examine a motivating problem illustrating these issues...
Objective: adjust fuel flow (manipulated var.) to keep gasoil outlet temperature at setpoint (controlled var.), despite variations in feed flow (disturbance var.).

Controlled variable (y): Temperature

Disturbances (d): Inlet Water Flows, Temperatures

The presence of “transportation lag” adds delay to the response of this system

System identification problem: generate the dynamical model relating changes in hot water flow to changes in shower temperature

A graphical system identification technique resulting from analyzing the output response \( y(t) \) to a step change of magnitude \( A \) in the independent variable \( u(t) \), introduced at time = 0.

\[
\tau \frac{dy}{dt} + y = Ku(t - \theta)
\]

Consider the presence of an integrated disturbance affecting temperature; the disturbance is shown here in the “open-loop” (i.e., valve position is held constant, and no changes are made to counteract the drift).
Compare Step Responses: FOPDT Model[-], PLANT Data[-], TRUE PLANT[-]

a close fit to the measured data leads to an erroneous model (with 25 to 30% error in all model parameters) as a result of the drifting disturbance.

How can we remedy this situation?

• Apply a systematic approach to dynamical modeling that relies on system identification methods.
• Design variable selection issues include (but not limited to):
  - Input signal design: random binary, pseudo-random binary (PRBS), or multisines? For how long, and at what amplitude?
  - Data preprocessing: detrending, control-relevant prefiltering, outlier removal, etc.
  - Model structure selection and parameter estimation: ARX, ARMAX, Output Error, Box-Jenkins. What orders to pick?
  - Model validation: simulation, cross-validation, correlation analysis, pole-zero plots, step and impulse responses.

Error in System Identification

Error = Bias + Variance

- Bias: Systematic errors caused by:
  - input signal characteristics (e.g., degree of excitation),
  - choice of model structure,
  - mode of operation (e.g., closed-loop versus open-loop).
- Variance: Random errors introduced by the presence of noise in the data, which do not allow the model to exactly reproduce the plant output. It is affected by the following factors:
  - number of model parameters
  - duration of the identification test
  - signal-to-noise ratio

“Shower” example using a PRBS input and PID with filter controller.
Course Outline

- Signals and systems overview.
- Nonparametric estimation and input signal design.
- Parametric model estimation and classical model validation.
- Control-relevant and closed-loop identification.
- Multivariable system identification (includes state-space).
- Issues in nonlinear and semiphysical modeling.

Course Focus

- Very broad subject
  - Linear or nonlinear? (Mostly) Linear
  - Continuous or discrete-time: Discrete-time
  - Parametric or nonparametric: Both
  - Time or frequency domain: Both

System Representations

Sampled Data (Discrete-Time) Systems

Nonparametric

Parametric
Pulse Transfer Functions

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Discrete-Time Signals and Systems

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System Identification Structure

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Signals Overview

- Deterministic versus stochastic signals.
- Stationary versus nonstationary signals.
- White versus autocorrelated signals.
- Crosscorrelated versus uncorrelated signals.

Mean, auto- and cross-covariance, power and cross-spectra will be among the measures utilized here.

The variance between $x(t)$ and $x(t + k)$ is called the autocovariance at lag $k$.

$$\gamma(k) = \text{cov}(x(t), x(t + k)) = E[(x(t) - \mu)(x(t + k) - \mu)]$$

Theoretical Autocovariance

$$c(k) = \frac{1}{N} \sum_{t=1}^{N} (x(t) - \bar{x})(x(t + k) - \bar{x})$$

Sample Autocovariance

Similarly the autocorrelation coefficient at lag $k$ is defined as

$$\rho(k) = \frac{\gamma(k)}{\sigma^2}$$

Theoretical Autocorrelation

$$r(k) = \frac{c(k)}{c(0)}$$

Sample Autocorrelation

Autocovariance describes the relationship between the same time series.

Crosscovariance describes the relationship between two different time series.
Correlation Analysis (CRA):

- direct estimation of impulse response coefficients from identification data.

Spectral analysis (SPA, ETFE):

- direct estimation of frequency responses from identification data.

Correlation analysis can be used to estimate the Finite Impulse Response (FIR) coefficients $b_0, \ldots, b_n$ between a measured input signal $u(t)$ and a measured output signal $y(t)$:

$$y(t) = b_0 u(t) + b_1 u(t-1) + b_2 u(t-2) + \ldots + b_n u(t-n_b) + \nu(t)$$

$\nu(t)$ is an unmeasured disturbance signal. The main assumptions behind correlation analysis are:

- stationarity of the time series $u(t)$ and $y(t)$ (if nonstationary, data pre-processing procedures can be applied to both $u(t)$ and $y(t)$ until stationarity is obtained),
- statistical independence between $u(t)$ and the noise $\nu(t)$, (i.e., $u(t)$ and $\nu(t)$ are uncorrelated),
- impulse response coefficients past $n_b$ are zero (i.e., $n_b$ must be chosen sufficiently high),
- persistence of excitation in $u(t)$ corresponding to the value for $n_b$.

Sample data set included as part of Matlab’s System Identification toolbox;

Power supplied to the heating device is the input $u(t)$; the output $y(t)$ is the outlet air temperature. Input signal corresponds to a Random Binary Sequence (RBS).

40 impulse response coefficients calculated for this data set;
The power spectral density $\Phi_x$ of a time series $x(t)$ is obtained by taking the Fourier transform of the autocovariance function

$$\Phi_x(\omega) = \sum_{\tau=\infty}^{\tau=-\infty} \gamma_x(\tau) e^{-j\tau\omega}$$

$\Phi_x$ is a function of frequency, $\omega$. In a spectral context, persistence of excitation of order $n$ signifies that $\Phi_x \neq 0$ at $n$ distinct frequencies.

Parseval’s Theorem establishes a relationship between the variance of $x(t)$ and its power spectrum

$$\gamma_x(0) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E[x^2(t)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_x(\omega) \, d\omega$$

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**Input Signals to Consider**

- Step and pulse inputs
- White noise
- Random Binary Signals
- Pseudo-random binary signals
- Multi-level pseudo-random signals
- Multisines (e.g., Schroeder-phased and minimum crest factor)

“Plant-friendliness” during experimental testing will be an important design consideration...

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**Pseudo-Random Binary Sequence (PRBS)**

- The PRBS is a periodic, deterministic input which can be generated using a shift register algorithm and Boolean algebra,
- The main design variables are the number of shift registers ($n_r$), the switching time ($T_{sw}$), the signal amplitude ($a$), and the number of cycles.
One cycle of the PRBS time input signal

$u(t)$

Time [Min]

-1
-0.5
0
0.5
1

Power Spectrum of the PRBS input

$\Phi_u(\omega)$

Radians/Min

-10
-5
0
5
10

- One cycle of a PRBS signal with $n_r = 4$ shift registers, switching time $T_{sw} = 2.1$ min, and signal amplitude $a = \pm 1$. The PRBS signal repeats itself every 31.5 min.

- The CSEL group has developed guidelines for PRBS design based on time constants and other system information available a priori.

A PRBS cycle can be repeated as many times as needed to create a data set with a desirable length.

- In the prediction problem, current and previous plant measurements are used to obtain estimates for $k+1$ (or beyond) time steps in the future.

- Assume that $y(t)$, $u(t)$, and $\nu(t)$ are stationary (or “quasi-stationary”) as before. $a(t)$ is white noise;

- Our goal is to obtain $\hat{p}(q)$ and $\hat{\nu}(q)$ as estimates for $p(q)$ and $H(q)$, respectively.

- $\hat{\nu}(q)$ is commonly referred to as the “noise” model. $r(t) = y(t) - \hat{y}(t|t-1)$ is the one-step-ahead prediction error.
The general family of prediction-error models corresponds to

\[ A(q)y(t) = \frac{B(q)}{F(q)}u(t-nk) + C(q)\tilde{e}_t(q) \]

\[ A(q) = 1 + a_1q^{-1} + \cdots + a_{nk}q^{-nk} \]
\[ B(q) = b_1 + b_2q^{-1} + \cdots + b_{nk}q^{-nk+1} \]
\[ C(z) = 1 + c_1z^{-1} + \cdots + c_{nk}z^{-nk} \]
\[ D(z) = 1 + d_1z^{-1} + \cdots + d_{nk}z^{-nk} \]
\[ F(z) = 1 + f_1z^{-1} + \cdots + f_{nk}z^{-nk} \]

which can be written in transfer function form as

\[ y(t) = \tilde{p}(q)u(t) + \tilde{\varphi}(t) \]
\[ \tilde{p}(q) = \frac{B(q)}{A(q)F(q)}q^{-nk} \]
\[ \tilde{\varphi}(t) = \frac{C(q)}{A(q)B(q)} \]

The AutoRegressive with eXternal input (ARX) model is represented via the

\[ y(t) \]
\[ y_{di} \]

The one-step ahead predictor for \( y \)

\[ \hat{y}(t-1) = -a_1y(t-1) - \cdots - a_{nk}y(t-nk) + b_1u(t-nk) + \cdots + b_{nk}u(t-nk-nk+1) \]

can be expressed as a linear regression problem via

\[ \varphi = [y(t-1) \ldots y(t-nk)u(t-nk) \ldots u(t-nk-nk+1)]^T \]

and \( \theta \), the vector of parameter estimates:

\[ \theta = [a_1 \ldots a_{nk} b_1 \ldots b_{nk}]^T \]

Rewriting the objective function as

\[ \min_{\theta} V = \min_{\theta} \frac{1}{N} \sum_{i=1}^{N} [y - \varphi^T(t)\theta]^2 \]

leads to the well-established ordinary least-squares solution

\[ \hat{\theta} = \left[ \frac{1}{N} \sum_{i=1}^{N} \varphi(t)\varphi^T(t) \right]^{-1} \frac{1}{N} \sum_{i=1}^{N} \varphi(t)y(t) \]
Understanding Bias

True Plant: \[ y(t) = p(q)u(t) + H(q)a(t) \]
Plant Model: \[ y(t) = \tilde{p}(q)u(t) + \tilde{p}_s(q)e(t) \]

Consider prefiltered input/output data
\[ y_f(t) = L(q)y(t) \quad u_f(t) = L(q)u(t) \]

Asymptotically (as the number of observations \( N \to \infty \)), the least-squares estimation problem can be written as:
\[
\min_{\tilde{p}, \tilde{p}_s, N} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} e_i^2(t) = \min_{\tilde{p}, \tilde{p}_s} \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{e,x}(\omega) d\omega
\]

where \( \Phi_{e,x} \) is the prefiltered prediction-error spectrum
\[
\Phi_{e,x}(\omega) = \frac{|L(e^{j\omega})|^2}{|\tilde{p}_s(e^{j\omega})|^2} \left( |p - \tilde{p}|^2 \Phi_a(\omega) + 2\text{Re}\left((p - \tilde{p})H^*(e^{j\omega})\Phi_{au}(\omega)\right) + |H(e^{j\omega})|^2 \sigma_n^2 \right)
\]

Understanding Variance

Model Parameter Variance \( \sim \frac{n}{N} \left( \frac{\sigma_p}{\sigma_u} \right)^2 \)

- \( n \equiv \) no. of model parameters
- \( N \equiv \) no. of data points (length of data set)
- \( \sigma_p^2 \equiv \) variance of the disturbance signal \( \nu(t) \)
- \( \sigma_u^2 \equiv \) variance of the input signal \( u(t) \)
- \( \sigma_u/\sigma_\nu \equiv \) input signal-to-noise ratio

- Model parameter variance increases with increasing model order and disturbance variance; increasing the length of the data set and input signal variance will lower parameter variance.

Consistent PEM estimation

Consistent prediction-error estimation, i.e., as \( N \to \infty \)
\[
\min_{\tilde{p}, \tilde{p}_s} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} e^2(t) = \sigma_n^2
\]

\( \tilde{p}(q) \to p(q) \quad \tilde{p}_s(q) \to H(q) \) with probability 1

is achieved when the following are true:
1. The model structure for \( \tilde{p}(q) \) and \( \tilde{p}_s(q) \) describes the true plant. A suitable model structure must be selected.
2. \( u(t) \) shows persistent excitation. The autocovariance matrix \( \Gamma_u \) for the input signal \( u(t) \) must be of full rank for dimensions corresponding to the order of the models \( \tilde{p} \) and \( \tilde{p}_s \). An equivalent statement is that the power spectrum of \( u(t) \) have nonzero power (\( \Phi_u(\omega) \neq 0 \)) over the number of frequencies corresponding to the model order.

Note: The theory does not require \( u(t) \) and \( a(t) \) to be uncorrelated sequences (i.e., \( \rho_{au}(k) = 0 \) for all \( k \)); however, if the input and the disturbance are uncorrelated, \( p(q) \) can be consistently estimated by \( \tilde{p}(q) \) despite an erroneous model structure for \( \tilde{p}_s(q) \). This has practical benefits.
Classical Model Validation

- Simulation (plot the measured output time series versus the predicted output from the model)
- Crossvalidation (simulate the predicted output on a data set different than the one used for parameter estimation)
- Information criteria (Akaike, Rissanen’s Minimum Description Length)
- Evaluate the model’s impulse, step, and frequency responses. Compare these with the results of correlation analysis, and determine if these agree with physical intuition.
- Residual analysis: perform correlation analysis on the one-step ahead prediction error - make sure that it resembles white noise.

ARX Model Structure Selection using Crossvalidation

Because ARX-\([n_a \ n_b \ n_c]\) estimation is accomplished using ordinary least squares, it can be applied exhaustively to a large number of model structures, with crossvalidation then used to determine a suitable model order.

Matlab System Identification Toolbox

Graphical User Interface shown; command-line functionality also available.

Interactive System Identification Demonstration Tool (ITSIE)

Written using Sysquake (www.calerga.com) in collaboration with Professors José Luis Guzmán and Manuel Berenguel Soria (Univ. of Almería, Spain) and Sebastián Dormido Bencomo (Spanish National Distance Learning University, Madrid).
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Modeling Requirements for Control

"Decomposed"  "Integrated/Synergistic"

Modeling → Modeling/Control
Control

Same result is not obtained from both approaches!

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Control-Relevant Identification

- Some general ideas behind control-relevant modeling.
- Design variables for control-relevant identification.
  - control-relevant prefiltering
  - control-relevant input signals
- Integrated system identification and PID control.
- Uncertainty estimation from identification data (if time permits).

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Control-relevant Prefiltering

- The purpose of control-relevant prefiltering is to emphasize information in the data that is most important for control design; illustrated above on the Shell phenol plant data.

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Closed-loop identification

- Problems will occur as a result of
  - crosscorrelation between disturbance and inputs
  - controller bias that “eats away” at input excitation
- Design choices include signal injection point and spectrum, and controller tuning during experimental testing.

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• Motivation for multivariable identification.

• Multiple input extensions to:
  - PRBS, RBS, and multisine design
  - ARX and PEM estimation

• Overviews of Bayard’s, Zhu’s, and subspace methods.

• Overview of ASU’s control-relevant multivariable methodology.

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• Multi-input PRBS design implemented on Weischedel-McAvoy high purity distillation column model.

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• Nonlinear identification is to system identification what “non-elephant” zoology is to the field of animal behavior (paraphrased from M. Deistler at TU-Vienna).

• The topic has to be examined in the context of specific model classes (e.g., Hammerstein/Wiener, Volterra, NARMAX).

• In general:
  - there are additional considerations in input design, parameter estimation, and validation that must be addressed.
  - parameter estimation can still be linear in the parameters, even if the model structure itself is nonlinear.

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• Some popular texts on system identification:

• Very broad reference textbook on discrete-time dynamical models

• Good reference on input signal design

• Identification and control, from a primarily statistical perspective

• To access some recent papers from CSEL on system identification:
  http://www.fulton.asu.edu/~csel/Publications-RecentSysId.htm